Exercise 6 / TKP4105 Håkon Olvik

1a) We have the energy balance

$$\frac{dH}{dt} = H_{in} - H_{out}$$

We know that

$$H = mC_vT$$

and that

$$H_{in} - H_{out} = Q - UA(T - T_L)$$

Thus

$$\frac{d}{dt}(mC_vT) = Q - UA(T - T_L)$$

Rewriten with regards to $\frac{dT}{dt}$

We get

$$\frac{dT}{dt} = -\frac{(T - T_L)}{\frac{mC_v}{UA}} + \frac{Q}{mC_v}$$

From the appendix we have

$$\frac{dy}{dt} = -\frac{y}{\tau} + b$$

This gives us that

$$\underline{\underline{\tau = \frac{mC_v}{UA}}}$$

b) $m = 1\,kg$, $A = 0{,}04\,m^2$, $U = 100\,W/m^2K$, $c_v = 0{,}5\,kJ/kgK$

$t = 0$ $T_L = T = 300\,K$ $Q: 0 \rightarrow 1\,kW$

$$\tau = \frac{mC_v}{UA} = \frac{1 \cdot 0{,}5}{100 \cdot 0{,}04} = \underline{125\,s}$$

c) Without control

$Q: 0 \to 1\,kW$ at $t=0$

from appendix:

$$y = y_0 e^{(-t/\tau)} + b\tau\left(1 - e^{(-t/\tau)}\right)$$

$$\Rightarrow T = T_0 e^{(-t/\tau)} + b\tau\left(1 - e^{(-t/\tau)}\right)$$

Also from appendix

$$\Delta y(t) = \Delta y(\infty)\left(1 - e^{(-t/\tau)}\right)$$

$$\Rightarrow \Delta T(t) = \Delta T(\infty)\left(1 - e^{(-t/\tau)}\right)$$

appendix

when $t \to \infty \Rightarrow 1 - e^{(-t/\tau)} = 1$

when $t \to 0 \Rightarrow 1 - e^{(-t/\tau)} = 0$

~~We have that~~

$$\cancel{\Delta T(\infty) = T(\infty) - T(0) = b\tau - T_0 = \frac{Q}{mC_v}\tau - T_0 = \frac{1}{1 \cdot 0{,}5} \cdot 125 - 300}$$

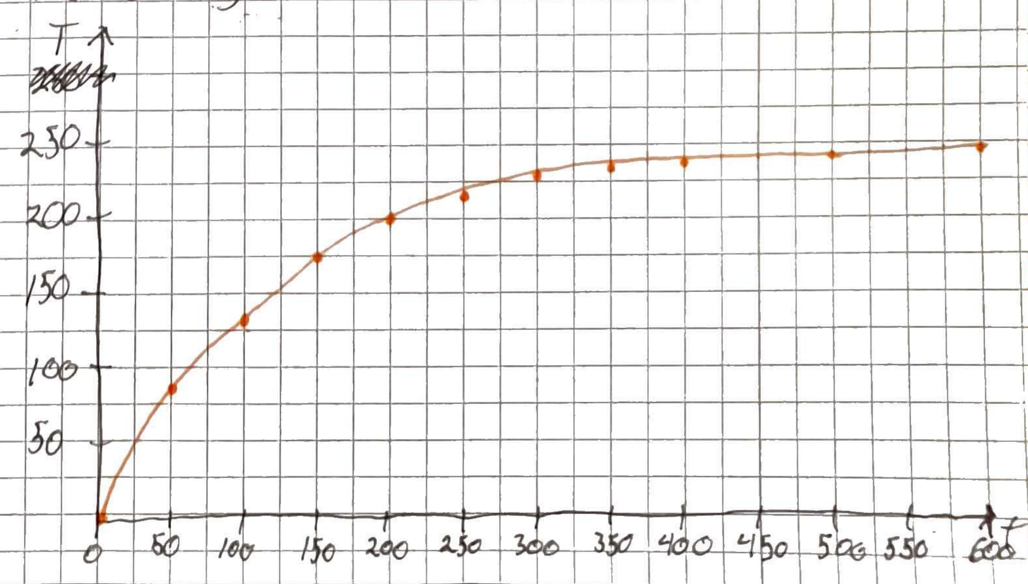
$$\cancel{\frac{kW\,s^{-1}}{kg\;kJ/kgK} \cdot s - K}$$

From studass: $Q = UA\Delta T$

$$\Delta T = \frac{Q}{UA} = 250K$$

$$\Delta T(t) = 250K\left(1 - e^{(-t/\tau)}\right)$$

t	$\Delta T(t)$
50	82,4
100	137,7
150	174,7
200	200,0
250	216,2
300	227,3
350	234,8
400	239,8
500	245,4
600	247,9
0	0



d) With control (Det er tøft å vɑre hubert) :)